Appendix A – Snapshot of Abstract Algebra – Professor Gold

Dr. Gold sits on the edge of an instructor's table in front of a whiteboard. Twenty students are seated among 36 small desk-chairs arranged in 6 rows of 6, all facing the front of the room. It is the first day of class.

Gold (putting the course syllabus aside): All right. Well, I think we should do some math. How many people are there here? Looks like about *(pointing at students and whispering numbers as he looks around the room)* 20 people. This is great. Let's split up into 5 groups. And I will tell you how: I will ask you to count up to 5 and then go back to 1. And you will remember your number. And you'll go to one of these big surfaces *(gestures to whiteboards circling the room)*. And you will write your names down, introduce yourselves, and then I'm going to give you a math problem to do. And then I'll give you more math problems to do. And then after that, there'll be more math problems *(some students chuckle)*.

(Dr. Gold establishes groups and asks students to move to their board areas. Groups have 5 minutes to introduce themselves and write names on the board. Gold walks to each group, handing out markers so each student has a marker, then stands in the center of the room.)

Gold: Okay, my friends, it is now time to talk about triangles. You all know what a triangle is, right? Three sides, three angles *(students smile and nod)*. Imagine holding a triangle in your hand so you can turn it all around and flip it over and over *(gestures rotating and turning)*. Draw me a picture of a triangle, and think about all the possible ways you can turn that triangle around and flip it over. Draw something that represents all those ways. Be prepared to tell me why your picture has the look that it does. Come get me when you feel like you've got a good picture.

(Students talking, writing on whiteboard. Gold walks around silently, looking at their work)

Gold (to Group 3, whispering): How's it going? You are all being really quiet.

Jaime (laughing): Yeah.

Gold (looking at the board): So, is this everyone's diagram?

Jaime: Yeah. We agreed that you could turn it all the way around and turn it over once.

Gold: Do you all endorse that picture?

Aleta: Sort of.

Gold: Sort of?

Aleta: Well, when we thought about turning the triangle, we thought about a thumbtack in the middle...well, middle-ish. Then, we could turn it to wherever we want it to go *(puts hand by the board and rotates hand to indicate rotating the sketched scalene triangle)*. But we didn't think of it that way to flip it.

Gold: How did you think about it?

Aleta: If the triangle wasn't attached to anything, we could just pick it up and turn it over. But what if we put our thumbtack here, here or here *(points to each of the three vertices of the triangle)*? Then, the way we could flip it over changes.

Gold: Do you want your triangle attached? Or do you want it unattached?

Aleta: It depends.

Gold: It depends on what?

Jaime: It depends on how you want to define "flip." Do you want to just turn the triangle over? Or do you want to see what it looks like turning it over on each vertex?

Gold: So, you've changed your mind? Or are you just messing with me? *(Laughs)* Okay. Well, why don't you all agree on something, and as long as you can tell a story that justifies it, I'm not going to freak out. Okay?

Jaime: Okay. *(Gold walks to Group 1)*

Gold: Hello, my friends. How's that picture looking? Is this everybody's picture? *(Group 1, has drawn an equilateral-looking triangle with arrows curving around each of the edges.)*

Katie: Yes.

Gold: So, I can ask anyone to tell me what the story of this picture is and you'll all be fine with that?

Sarah: Yep.

Gold (turning to Blake): Okay, how about you tell me why this picture has this look?

Blake: Well, we drew our triangle and we noticed that if we started turning it, the points could all land on the points exactly. So there are three turns.

Gold: Do you all agree with that? Are you all on the same page about this interpretation? *(Turning to Avi)* Why don't you tell me about what happens when you flip the triangle?

Avi: Um. I…um…don't know how to talk about it. It will…um. I think it…

Gold: Why don't I come back? Just make sure you all get on the same page and have some sort of explanation. I mean, it looks like a pretty good picture. Just get the words right. I'll be back in a minute.

(Gold visits Groups 2 and 5, gives them the handout, revisits Group 3, returns to Group 1)

Gold: Do you have it all cleared up? Tell me the story because I like that picture. Can you *(to Avi)* get it started and then we'll go to other people?

Avi: Well, we decided that you pick it up and flip it over. And there's only one way to do that.

Gold: Okay. Did you understand it before and just couldn't find the words?

Avi: I didn't understand it clearly.

Gold: But you understand it now? *(Avi nods)* And you all agree with that? *(Students nod)* Well, I think it's important to make sure you're all on the same page. And we might have to work on language, math language, whether or not English is our first language. But that's what happens. And I don't want to make people feel bad. But I just want to make sure that everyone's communicating. Because otherwise, it's easy to just say, "Yeah, I sort of get it, I'll just see what happens." *(Students nod)* So, let me give you something else to think about. *(Gold gives each a handout, with prompts: (1) Given a triangle, describe the set of rotations that leave the triangle fixed. (2) Now add the reflections to the triangle to this set. Describe the set.)*

Gold (approaching Group 4 after 15 minutes of work): This is a nice picture. Can you all explain it to me? *(Students nod)* So, I could ask any of you to tell me about it, and you'd all be pumped? *(Three students nod)* Okay, great! *(Looking at fourth student)* Why don't you tell me the story of this picture?

Daylen: We decided "leave the triangle fixed" meant whatever we do to it, it has to look like the original triangle. And the only way for that to happen is to have an equilateral triangle *(others nod)*. So, we started messing with it and labeled the points *A*, *B*, and *C* so we could stay organized. Then we could say this rotation *(Puts hand by the board and rotates it to indicate rotating the triangle clockwise by one-third)* makes it *BCA*, and this rotation *(puts hand by the board and rotates clockwise by two-thirds)* makes it *CAB*. And then we reflected three times using each point and the middle of the opposite side.

Gold: Okay. Do you all agree with that explanation?

John (laughing): Is that the right answer?

Gold: I thought it was a *good* answer. I think it's time to go whole class.

(Gold walks to center of the room, students turn to see him, some are talking)

Gold: Okay, my friends, it is that special time of the class when we talk together as one group. Who would like to tell me…I know it's hard to stop talking sometimes. We're now at whole-class conversation *(Gold stops talking and waits until it is quiet).* I want to debrief. First, look around at what people drew. And you can privately think about whether you think people are crazy or whether you agree with the pictures they drew. *(Gold pauses for 1 minute)* Do you see all these interesting shapes? There's some really out-there ones. I mean that in a good way. I'll take a few hands, if anyone wants to tell me about a picture they see. *(Several people begin talking.)* Wait, wait, wait, I'll call on you in a second. But I want to see some hands from some people who would like to comment on this. Okay, great. I see two hands. Anyone else have an opinion? I'd like to see a few more. Think about it for 10 seconds. *(Waits 10 seconds.)* Okay, can I see some hands from anyone who has some ideas about these diagrams? Okay, I see four hands. Let's go around *(Gestures clockwise around the room, starting with the student closest to his left. Whole group conversation continues for 10 minutes with students raising their hands and describing and justifying what they see as "reasonable" and "unreasonable" representations).*

Gold: Okay. That's about all of the time we have today. That was a really great conversation. I really liked how you disagreed with each other respectfully and you tried to use specific language about your pictures. And actually, all these things we've talked about today have really specific mathematical language. We'll start digging into that next class. Secondly, you may have noticed I went around to the groups and kind of needled you a little bit to explain your thinking. That's not because I don't like you. Part of the reason I'm doing this is that I want to give you a chance to use the language of mathematics to argue. So, when I come around and bug you about things – like when I ask, "Why is this true?" Or "can every person in the group explain?" – it's not because I'm suspicious of you and it's not a hint that you're wrong. The reason is that I want you to use the language of math to explain. And the reason I keep on bugging you to keep each other together is that there's not really a point to doing this in groups if you're not going to work together. And what do you get out of working as a team? I don't care whether you get the right answer or not, to be honest, unless you're able to convince each other why it is a worthwhile answer.